Lattice QCD thermodynamics with improved dynamical fermions

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in collaboration with

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Outline

1. Introduction
2. QCD and its lattice formulation
3. Transition temperature
4. Equation of state
5. Phase diagram for small densities
Motivation

High temperature field theory
  – Early Universe
  – High energy collisions
(Phase) transitions
  – Electroweak transition
    particles acquire mass via Higgs mechanism
    \( \rightarrow \) no real transition
  – QCD transition
    protons/neutrons form, become massive
order/temperature of transition?

Non-zero density
  – neutron stars
  – heavy ion collisions
Chromodynamics is a generalized electrodynamics

QCD is a generalized, extended version of QED

**Electrodynamics**: only 1 charge, electric (positive or negative)

**Chromodynamics**: 3 charges, call them colors (red, blue, green) all of them can be positive or negative (not real colors)

Gluons (similarly to photons) transmit the strong interaction between quarks (which are similar to electrons)

Richer structure: color of quarks can be changed by gluons (charged)

Neither free gluons nor quarks have ever been seen

Fundamental degrees of freedom don’t appear experimentally
Quantizing field theory

The basic tool to understand particle physics:

quantum field theory

field variables, e.g. \( A_\mu(\vec{r}, t) \), are treated as operators

\( \Rightarrow \) particles e.g. photons
(moving energy packages with some definite quantum numbers)

symmetries + internal consistency fix the Lagrangian

\( \Rightarrow \) unambiguously fixes the interactions between particles
at LEP the process can be clearly seen ($\approx 1\%$ of the QED processes) this is the only elementary process in QED
Great success of the perturbative approach

the strength of the elementary process is small ≈ 1/137
precision perturbative predictions: magnetic moment of $e^-$

upto 13 digits complete agreement with experiments

$$\mu_e = 2.00231930443622(14), \text{ experiment}$$
$$\mu_e = 2.0023193044352(16), \text{ theory}$$
Basic interaction(s) in QCD

quark emits a gluon
(or a gluon emits one/two gluons)

at LEP the process can be clearly seen ($\approx 10\%$ of QCD processes)
we see jets and verify the underlying equations: asymptotic freedom
we do not see free quarks or gluons: confinement phenomena
Running of the strong coupling, asymptotic freedom

electric charge is screened:
at small distances (large momenta) we see “more and more charge”

color charge is anti-screened:
at small distances (large momenta) we see “less and less charge”
coupling is getting smaller

D. Gross, F. Wilczek, D. Politzer ’73
Nobel Prize 2004

dozens of experiments prove asymptotic freedom
at large distances (small energies) the coupling is large: confinement
QCD: need for a systematic non-perturbative method

in some cases: good perturbative convergence; in other cases: bad pressure at high temperatures converges at $T=10^{300}$ MeV
Lattice field theory

systematic non-perturbative approach (numerical solution):

quantum fields on the lattice

quantum theory: *path integral* formulation with $S=E_{\text{kin}}-E_{\text{pot}}$

quantum mechanics: for all possible paths add $\exp(iS)$
quantum fields: for all possible field configurations add $\exp(iS)$

Euclidean space-time ($t=i\tau$): $\exp(-S)$ sum of Boltzmann factors

we do not have infinitely large computers $\Rightarrow$ two consequences

a. put it on a *space-time grid* (proper approach: asymptotic freedom)
   formally: four-dimensional statistical system
b. *finite size of the system* (can be also controlled)

$\Rightarrow$ stochastic approach, with reasonable spacing/size: solvable
fine lattice to resolve the structure of the proton ($\lesssim 0.1$ fm) few fm size is needed.

50-100 points in ‘xyzt’ directions $a \Rightarrow a/2$ means 100-200 $\times$ CPU

mathematically

$10^9$ dimensional integrals

advanced techniques, good balance and several Tflops are needed.
The nature of the QCD transition

finite size scaling study of the chiral susceptibility

$$\chi = \frac{(T/V) \partial^2 \log Z}{\partial m^2}$$

similar to a magnetic susceptibility

first/second order phase transition: susceptibility diverges

**phase transition**: finite V analyticity $V \to \infty$ increasingly singular

(e.g. first order phase transition: height $\propto V$, width $\propto 1/V$)

for an **analytic cross-over** $\chi$ does not grow with $V$

two steps (three volumes, four lattice spacings):

a. fix $V$ and determine $\chi$ in the continuum limit:

b. using the continuum extrapolated $\chi_{max}$: finite size scaling
The nature of the QCD transition: result

finite size scaling of the inverse susceptibility

![Graph showing finite size scaling of the inverse susceptibility.]

The result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range chance probability for $1/V$ is $10^{-19}$ for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

the QCD transition is a cross-over
Smooth analytic transition (cross-over)

Hot Aging of the Universe Cold
### Transition temperatures for various observables

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- **chiral condensate**
- **quark number susceptibility**

**Equation of state**

**Phase diagram for small densities**

**S. D. Katz**
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Equation of state

- Two lattice spacings ($N_t = 6, 8$) + checkpoints ($N_t = 10, 12$)
- nice scaling
- everything is derived from the pressure
The QCD phase diagram

non-zero chemical potential $\rightarrow$ sign problem
Monte-Carlo based on importance sampling fails

We can still calculate derivatives at $\mu = 0$
Phase diagram for relatively small $\mu$ can be given